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ABSTRACT

This paper presents some findings on students and authority in the mathematics classroom. It is shown that students create a web of authorities that extends not only to their teachers but also to their fellow students. While it is not asserted that relationships based on authority should be abolished, it is shown that such relationships can interfere with the formation of intellectual partnerships necessary, for example, in collaborative learning. It is also suggested that the dominance of authority relationships may hinder the development of reflective thinking about mathematical ideas. A brief description is also given of the Learners Perspective Study, in which these observations were made. (Author)

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AUTHORITY IN THE MATHEMATICS CLASSROOM AND ITS INFLUENCE ON STUDENTS' ABILITY TO REFLECT

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This paper presents some findings on students and authority in the mathematics classroom. It is shown that students create a web of authorities that extends not only to their teachers but also to their fellow students. While it is not asserted that relationships based on authority should be abolished, it is shown that such relationships can interfere with the formation of intellectual partnerships necessary, for example, in collaborative learning. It is also suggested that the dominance of authority relationships may hinder the development of reflective thinking about mathematical ideas. A brief description is also given of the Learners Perspective Study, in which these observations were made.

Much research in recent years has been dedicated to the social context of mathematics education (e.g. Clarke, 2001; Cobb & Bauersfeld, 1995; Edwards & Mercer, 1987). Indeed, it has become almost a truism that understanding the social context of learning is essential to understanding mathematical learning itself. Thus, there is good reason to study students' relationship to authority: who becomes authority figures for students, what their authority means to students, and how does the relationship to authority affect learning. These are the big questions towards which this paper hopes to make some small steps.

THE RESEARCH SETTING AND METHODOLOGY: THE LEARNERS' PERSPECTIVE STUDY

The considerations on students and authority presented in this paper emerged from a more extensive, and still ongoing, study of the students' point of view, called the Learners' Perspective Study (Clarke, 1998, 2000). The goal of that study is to explore a number of questions concerning the ways students conceive mathematics classroom practice and mathematics learning. Since the answers to such questions are likely to have a cultural dimension, the project adopts an international approach, with research teams working simultaneously in Australia, Germany, U.S.A, Hong Kong, Japan, Israel, Sweden, and South Africa. The authors of the present paper are the principal investigators for the Israeli team.

The Learners' Perspective Study arose out of the Third International Mathematics and Science Study (TIMSS). TIMSS not only established national profiles of student achievement, but also sought to identify national norms for teaching practice that might account for "poor" or "high" achievement scores by videotaping and analyzing a statistically representative sample of eighth-grade mathematics classes in Japan, Germany and the USA (Fernandez, C. et al., 1997). Although this component of the TIMSS study was impressive and unprecedented in international comparative studies,

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the validity of the "effective scripts" discerned in the TIMSS videos was widely debated and not universally accepted (e.g. Keitel & Kilpatrick, 1999; Stigler & Hiebert, 1997, 1998, 1999). One of the major objections was that the TIMSS Video Study focused exclusively on the teacher and ignored the important role students have in the learning process. The present project, accordingly, overcomes this objection and expands on the work done in the TIMSS study by focusing on *student actions* within the context of whole-class mathematics practice and by adopting a methodology whereby student reconstructions and reflections are considered in a substantial number of videotaped mathematics lessons.

Two specific cases formed the basis for this paper. The first was a sequence of 15 lessons on systems of linear equations taught by a dedicated and experienced teacher, whom we shall call Danit. Danit teaches in a comprehensive high school. Her 8th grade class is heterogeneous and comprises 38 students, mostly native born Israelis, but also new immigrants from the former Soviet Union and one new immigrant from Ethiopia. The second case was a sequence of 4 lessons on geometry taught by a teacher, whom we shall call Sasha. Sasha is a new immigrant from the former Soviet Union with several years experience teaching in Israeli schools and much experience teaching in Russian schools. His 8th class is a high-level class and comprises 30 students.

As specified in Clark (2000), all the classroom sessions were videotaped using an integrated system of three video cameras, one viewing the class as a whole, one on the teacher, and one on a "focus group" of two or three students. Following each lesson, the students in the focus group were interviewed, and their notebooks, containing the notes for that particular lesson, were photocopied. Moreover, once a week the teachers themselves were interviewed. Although we had a basic set of questions for both the student interviews and the teacher interview, we allowed the interview protocol to remain flexible so that we could freely pursue particular classroom events; in this respect, our interview methodology was along the lines of Ginsburg (1997). An important aspect of the interviews was that during the interviews the students could view and react to the videotape of the lesson for which they were the focus group. Needless to say, the interviews themselves were also videotaped.

Among the research questions for the study originally set out in Clarke (2000) was the question of whether teacher and learner practices are conflicting or mutually sustaining. This led us to ask the students about the circumstances in which they request help from their teacher, and, from there, whether they request help from other people as well. These questions, among others, gave rise to a new set of questions for us concerning students and authority, namely, Who is an authority for students? What is the extent of the authority of various people? How pervasive is the influence of authority in students' mathematical lives? What effect does students' relationship with authority have on their mathematical practice? Our preliminary findings regarding these questions are the subject of this paper.

RESULTS

Students' Sources of Assistance

When students were asked to whom they turn for help when they run into difficulties, they provided always the following sources of help: their teacher, their friends, their parents, or their siblings. Of these, the students' teacher and friends were the dominant sources of help spoken about in the interviews. When asked to whom they turn *first*, some students said the teacher and some their friends. The reason given for turning to friends first was almost always that the "teacher is too busy and can't get to everyone." Often, however, it seemed to us from our observations of the class that students turned to their friends first simply because their friends were near, for usually they were students sitting at the same desk.

Teachers, friends, parents and siblings form a web of sources of assistance; when one source is unavailable or unable to help, one turns to another. For example, if Yarden in Sasha's class cannot get help from Sasha, for one reason or another, she turns to one of her friends:

Interviewer: Whom do you turn to for help with homework?

Paulina [the second girl in the focus group]: There's a "dialogue hour" with the teacher.

Interviewer: What if that's not possible?

Yarden: I call a friend.

Interviewer: And if your friend doesn't know?

Yarden: If my friend doesn't know, I ask someone else—or my father.

Can this web be characterized as a hierarchy? One sense in which it can has already been alluded to, namely, as a hierarchy according to *physical proximity*. Thus, in this hierarchy friends, since they sit at the same table, come first, then the teacher, then parents and siblings. However, more significantly, the web forms a hierarchy according to the *degree of authority* possessed by the sources. By "the degree of authority" we mean the degree to which a person's statements are to be taken unchallenged (this sense, in a way, is already built into the word "source"; indeed, the Greek word for "source," *arche*, also means "sovereignty," and in the plural, *hai archai*, "the authorities"). Conversely, turning to an authority means turning to a person for an answer or for instructions, not, by contrast, for a discussion. Moreover, we use the word "authority" rather than, say, "expertise" because the reason a person's statements are not to be challenged is, as we shall soon see, not always dependent on the degree of the person's knowledge, though it may be perceived that way. Now, in this hierarchy, there is no question, the teacher comes first.

The Teacher's Authority

The teacher's tremendous authority, in every sense of the word, was evident in all of the student interviews in both Danit's and Sasha's class. For example, at one point in

our interview with two students in Danit's class, Moshe and Sharon, we asked whether a graphical method or algebraic method of finding the solution to a system of equations was more reliable. Here is the exchange:

Moshe: If I get a answer for one and a different answer for the other, then you've got to check. If I get the same answer, then I'll believe it's correct. But if there's, maybe, still some doubt in my mind, I ask Danit.

Interviewer: What does Danit have that other people don't?

Moshe: She's a teacher, she can help; if you make a mistake, she corrects it!

Interviewer: And if she errs?

Moshe: She doesn't err.

Sharon: She studies everything at home before she comes to class.

Moshe: Otherwise she couldn't correct—she's a teacher!

Interviewer: But she did make a mistake at the board [during the lesson, Danit had made a careless error at the board].

Sharon: She got mixed up because she substituted wrong.

Moshe: Those are nonsense things she gets mixed up about, but real things [gestures to show the weightiness of the things he has in mind]—if two exercises [systems] are supposed to get the same answer or not, it doesn't seem to me she'd get mixed up about that.

In this exchange, one is impressed by the extent to which Moshe and Sharon are willing to see Danit as nearly infallible, and the extent to which they are willing to defend her authority, even when she is seen to make a mistake. The students view her, apparently, not only as one who knows more than they do, but also as a strong figure with powers they lack. When Moshe says, "She's a teacher, she can help; if you make a mistake, she corrects it!" he sounds as if he is speaking of a healer, a miracle worker, rather than of his 8th grade math teacher. Similarly, when we asked Sivan and Shimrit, also Danit's students, what exactly do they expect from the teacher when they ask her for help, Sivan said simply "That she will explain to us better," to which Shimrit added immediately, "When she comes over to me, when she explains to me, *suddenly I understand better*...[emphasis added]." Consistent with this image of Danit, was the importance the students seemed to place on the mere fact of Danit's coming over to help them when they worked on exercises. When we asked what the climax of the lesson was, Elah, in the same interview in which Sivan and Shimrit participated, answered, "When I was having trouble with the book and I called [Danit]." In a different interview, another girl in Danit's class, Gal, answered the same question in precisely the same way. Conversely, on two different occasions we came across a student in Danit's class who also appeared to be having trouble with the exercises, but who did not ask Danit for help. When we asked them why not, we received the same response both times: "The teacher doesn't want to help

me." Such a statement presents a picture in which the attention the students receive from the teacher is dependent on the teacher's whim. The teacher becomes, in this interpretation, a dictator, though, surely, for most students, a beneficent one who willingly helps them when they need help. Nevertheless, conceiving the teacher as a creature of whim is to conceive the teacher as a creature with terrific power.

The Web of Authorities

That the teacher should be given this degree of authority by the students is perhaps not very surprising. However, we were surprised to see how easily students are willing to see other people as authorities to a degree similar to that to which they see their teacher as an authority. For example, we were interested in seeing how students understood the significance of "showing their work," whether this was only a requirement of students or of mathematics itself. So, we asked whether a salesman who explained to customers how much they should pay given such and such a discount had to show his/her work. To this, Boaz, again from Danit's class, replied: "No, I can rely on him...I can rely on him—for sure lots of people come to him—there must be those who know percentages and things, and they rely on him, so I can rely on him too." It is worth noting here that the Hebrew word Boaz uses for "rely" is *somech*, which is closely related to the word *somchute* meaning, literally, "authority."

Students tend to see authorities at every turn. Their web of sources of assistance becomes, in this light, a true web of authorities. What is particularly striking, though, is that this extends also to the students' friends. As mentioned above, friends in the class are a dominant source of help. But when the students turn to their friends they tend to turn to them only for answers. And, as we saw with Sasha's student Yarden, when one friend does not know, she turns to another. In one interview in Danit's class, we asked a student why he did not ask his friend for help at a certain point during the lesson. He replied, "I knew Uri wouldn't know the answer..." Thus, when students are perceived by their fellow students as knowing the answer to some question *they are treated for that instant as an authority*, that is, the answer is accepted and not discussed. When students are not perceived as knowing the answer, they are usually not asked. In fact, in the classroom videos it can be seen quite often (though less so in the geometry classes) that students sit together, occasionally speak together, but do not really work together, even though they are not necessarily encouraged to work individually.

To understand the significance of this tendency of students to treat one another as authorities *ad hoc*, one needs first of all to see what alternative stands opposed to it. This need not be considered hypothetically, for in Sasha's class we found an exception to the tendency. During his lesson, Sasha gave a geometry problem to the class; we watched as two girls, Yulia and Roni, solved the problem in a truly collaborative spirit. Roni showed her diagram to Yulia; Yulia commented and pointed to her own diagram; they discussed the problem together, and, finally, came to a solution. Yulia and Roni happened to be our focus group for that lesson, and throughout the interview we saw how different their behavior was from other

students we observed: they consulted with one another, raised possibilities on their own, revised opinions, and seemed to arrive at common conclusions. In other words, rather than treating one another as possible authorities, that is, only as possible sources of answers, Yulia and Roni treated one another as intelligent interlocutors who could work together to make progress on the question at hand. We should stress that this was, indeed, behavior different not only from that of students in Danit's class, but also from that of other students in Sasha's class. For instance, at one point in our interview with Yarden, we asked if she could draw a triangle having two acute exterior angles; she said she could, and she proceeded to draw a diagram, which, obviously, could not be correct. When we asked Paulina, the second girl in the focus group, whether Yarden's diagram was ok, she assented immediately and with no further remark.

CONCLUSIONS—AUTHORITY AND STUDENTS' ABILITY TO REFLECT

Speaking about argumentation, Simon (2000) claims that, because students treat teachers as authorities who can give a stamp of approval on the students' mathematical arguments, it does not follow that students lack the ability and willingness to engage in the validation process themselves. This is a reasonable claim, and, certainly, our interview with Moshe, quoted above, gives some confirmation of it. Simon compares the students' relationship to the teacher, in her/his capacity as an authority, to the situation of a mathematician who is working in a field not her/his own and who seeks the evaluation of an expert in that field (Simon, 2000, p.166). But the evidence presented above shows that the way the teacher is an authority for the student is much stronger than this. In this connection, we tend to agree with Lewis-Shaw when she writes "[The students'] perspective consists in relating to the world with less awareness of the nature and extent of their personal authority or control. Consequently, they strive for a sense of belonging, count on the teacher for some level of security and authority and regard him as the holder of knowledge and expertise" (Lewis-Shaw, 2001, p.182).

This strong authority relationship is bound to effect the way students engage in mathematical thinking. An indication of this is given in Helme and Clarke (2001). They report that when a teacher approached a small group of students working on a problem there was a different pattern of interaction in which the teacher's questions became the central focus. Moreover, "With the teacher asking virtually all the questions, there was little opportunity for students to initiate ideas or spontaneously express and resolve uncertainty" (Helme & Clarke, 2001, p.146). Such a situation cannot be conducive to reflective thought about mathematical ideas.

In Danit's class, we observed an interesting instance that demonstrates, we believe, that the teacher's authority can create conditions in which it becomes very easy for students to use concepts unreflectively. At a certain stage while discussing the system,

$$\begin{cases} 2x+3y=18 \\ y=3 \end{cases}$$

Danit asked what allows us to go from 2x+3y=18 to 2·3+3y=18, that is, what allows us to substitute 3 in place of x? She prompted the class, saying, "It begins with 'A'..." Finally, Uriel, whom we interviewed afterwards, said "axiom." But which "axiom"? Danit gave the answer the "axiom of replacement." We asked Uriel about what went on in that instance. Uriel told us how he remembered the word "axiom" when Danit said that the word began with "A". We discussed the meaning of axiom, which he seemed to grasp in a very rudimentary way. However, when we asked Uriel if he had any idea why such an axiom might be needed, he had difficulty grasping what we were asking him. It became clear that Uriel recognized an axiom here mainly because Danit said there was one. In other words, the need for an axiom was accepted on Danit's authority only. This was striking because more than once during the lesson Danit emphasized that things in mathematics were not true because she said so, or the because the book said so, or because anyone else simply said so.

That the pattern of authority existing between teacher and student can also come to characterize the relationship between students, even in a small measure, clearly has important implications for collaborative learning. For as Johnson and Johnson say, "Simply placing students in groups and telling them to work together does not promote greater understanding of mathematical principles or the ability to communicate mathematical reasoning to others. Group efforts can go wrong in many ways" (Johnson & Johnson, 1989, p.237). Where students are accustomed to treat one another as authorities one student will simply listen and assent to the other, or, if not, will turn to a different authority, just as Yarden turns to another friend when the first "doesn't know." The point is, where students are accustomed to treat one another as authorities no true dialogue takes place between them, and where there is no true dialogue there can be no true collaborative learning. Thus, the establishment of an authority relationship between students becomes one of the most potent ways in which "Group efforts can go wrong"! The point would be merely an academic one if it were not that collaborative learning seems to be one of the more effective means of encouraging reflective thinking (e.g. Johnson & Johnson, 1989, p.236). In this way, by helping to develop a relationship between students not based on authority, but on intellectual partnership, we go far towards encouraging also the kind of thoughtful approach to learning mathematics that most of us strive for.

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